

# NOISE CANCELLATION OVER SPATIAL REGIONS USING ADAPTIVE WAVE DOMAIN PROCESSING

Jihui Zhang<sup>1,2</sup>, Wen Zhang<sup>1</sup>, Thushara D. Abhayapala<sup>1</sup>

<sup>1</sup> Research School of Engineering, The Australian National University,  
Canberra ACT 2601, Australia

<sup>2</sup>School of Electrical Engineering and Automation, Harbin Institute of Technology,  
Harbin 150001, China

## ABSTRACT

This paper proposes wave-domain adaptive processing for noise cancellation within a large spatial region. We use fundamental solutions of the Helmholtz wave-equation as basis functions to express the noise field over a spatial region and show the wave-domain processing directly on the decomposition coefficients to control the entire region. A feedback control system is implemented, where only a single microphone array is placed at the boundary of the control region to measure the residual signals, and a loudspeaker array is used to generate the anti-noise signals. We develop the adaptive wave-domain filtered-x least mean square algorithm. Simulation results show that using the proposed method the noise over the entire control region can be significantly reduced with fast convergence in both free-field and reverberant environments.

**Index Terms**— Active noise control, Adaptive algorithm, Feedback system, Multi-channel system, Wave domain signal processing

## 1. INTRODUCTION

Active noise control (ANC), or noise cancellation, employs secondary sound sources to reduce the primary noise based on the principle of destructive interference [1]. This technique has been successfully used in several commercial applications, such as noise-cancelling headphones [2], the control of noise in industrial machines [3] and active noise cancellation for automobiles [4]. Given the fact that most of the time noise is unknown and time-varying, normally adaptive filters are employed to produce anti-noise signals. Some well-known algorithms for implementation include the least-mean-square (LMS) method or its variants, such as filtered-x LMS (FxLMS), adjoint LMS and recursive LMS [5].

The technique of noise cancellation at some spatial points is simple and effective, where only one or two microphones and speakers are required to measure the residual signal and produce the anti-noise signal. A growing research interest is in creating a large-sized quiet zone for multiple listeners, which requires multiple microphones and loudspeakers to be used and is known as multi-channel ANC system [6]. Both time-domain [7, 8] and frequency-domain [9, 10] algorithms have been implemented in the multi-channel ANC systems. Especially, multi-channel version of FxLMS algorithm in the frequency domain has been widely used in practice [10, 11]. One drawback of the conventional multi-channel ANC

system is that to achieve a quite zone, error microphones need to be uniformly placed inside the region to measure the residual signals so that the noise energy within the entire region can be cancelled completely.

Recently, wave-domain signal processing has been proposed to achieve sound control over large spatial regions, such as echo cancellation [12–14], room equalization for massive multichannel sound field reproduction systems [15–18] and acoustic quiet zone generation [19–21]. The principle of wave-domain signal representation is to use fundamental solutions of the Helmholtz wave-equation as basis functions to express any wavefield over a spatial region. Processing directly on the decomposition coefficients therefore controls sound over a region. In [22], a wave-domain ANC system has been investigated, where one loudspeaker array and two microphone arrays, including one reference microphone array placed outside the loudspeaker array and one error microphone array placed at the boundary of the quiet zone, are adopted and the feedforward ANC system is implemented. Significant noise cancellation over the entire region of interest has been achieved.

In this paper, we propose the feedback ANC system in the wave domain using only a single error microphone array placed at the boundary of the desired quiet zone to measure the residual signals and one loudspeaker array to produce anti-noise signals (see Figure 1). Adaptive wave-domain FxLMS algorithm is implemented and demonstrates good performance for noise cancellation in both free-field and reverberant environments. The rest of this paper is organized as follows. Section 2 provides the system model of the wave-domain ANC system. In Section 3, we derive adaptive wave-domain processing algorithm. In Section 4, simulations are conducted to evaluate the proposed algorithm in comparison with the conventional multi-channel ANC system. Conclusions are provided in Section 5.

## 2. SYSTEM MODEL

The noise field due to a 2D point source at an arbitrary point  $\mathbf{x} \equiv \{\|\mathbf{x}, \phi_{\mathbf{x}}\|\}$  can be represented as

$$N(\mathbf{x}, k) = \sum_{m=-\infty}^{\infty} \beta_m(k) J_m(k\|\mathbf{x}\|) \exp(im\phi_{\mathbf{x}}), \quad (1)$$

where  $k = 2\pi f/c$  is the wavenumber,  $f$  is the frequency and  $c$  the speed of sound propagation,  $J_m(\cdot)$  is the Bessel function of order  $m$  and  $\exp(\cdot)$  denotes the exponential function. The wave domain coefficients  $\beta_m(k)$  represent the noise field.

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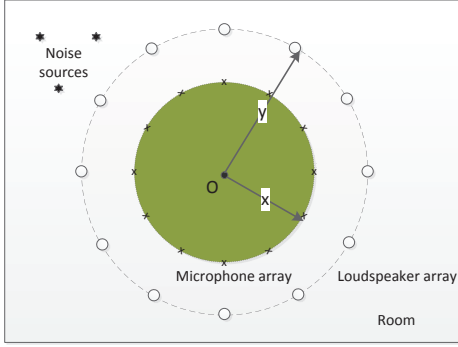


Figure 1: ANC setup with a circular quiet zone (green area) using a single microphone array to measure residual signals and a loudspeaker array to produce anti-noise signals.

The aim here is to cancel the noise completely within a circular region of interest, i.e., to create a quiet zone, using active control techniques as shown in Figure 1. Within this circular region  $\|\mathbf{x}\| \leq r$ , we can use a finite number of modes to approximate the noise field [23], that is (1) reduces to

$$N(\mathbf{x}, k) \approx \sum_{m=-M}^M \beta_m(k) J_m(k\|\mathbf{x}\|) \exp(im\phi_{\mathbf{x}}), \quad (2)$$

where  $M = \lceil ekr/2 \rceil$  [23, 24].

A circular array of  $L$  equiangularly placed loudspeakers is used to generate the control signals. We assume a continuous loudspeaker distribution  $\rho(\mathbf{y}, k)$ , where

$$\rho(\mathbf{y}, k) = \sum_{m=-\infty}^{\infty} \gamma_m(k) \exp(im\phi_{\mathbf{y}}). \quad (3)$$

Thus, the sound field generated can be written as

$$S(\mathbf{x}, k) = \int_0^{2\pi} \rho(\mathbf{y}, k) A(\mathbf{x}|\mathbf{y}, k) d\phi_{\mathbf{y}}, \quad (4)$$

where  $A(\mathbf{x}|\mathbf{y}, k)$  denotes the acoustic transfer function (ATF) between a source  $\mathbf{y}$  and an observation point  $\mathbf{x}$ . When the observation points are inside the quiet zone, the ATF can also be parameterised in the wave domain [16] as

$$A(\mathbf{x}|\mathbf{y}, k) = \sum_{m=-M}^M a_m(k) J_m(k\|\mathbf{x}\|) \exp(im\phi_{\mathbf{x}}), \quad (5)$$

where  $a_m(k)$  are ATF coefficients and assumed to be a prior knowledge obtained from theoretical solutions or pre-calibration.

Substituting (3) and (5) into (4) leads to the wave domain representation of the sound field generated by the control sources, that is

$$S(\mathbf{x}, k) \approx \sum_{m=-M}^M \gamma_m(k) \alpha_m(k) J_m(k\|\mathbf{x}\|) \exp(im\phi_{\mathbf{x}}), \quad (6)$$

where

$$\alpha_m(k) = \int_0^{2\pi} a_m(k) \exp(im\phi_{\mathbf{y}}) d\phi_{\mathbf{y}}. \quad (7)$$

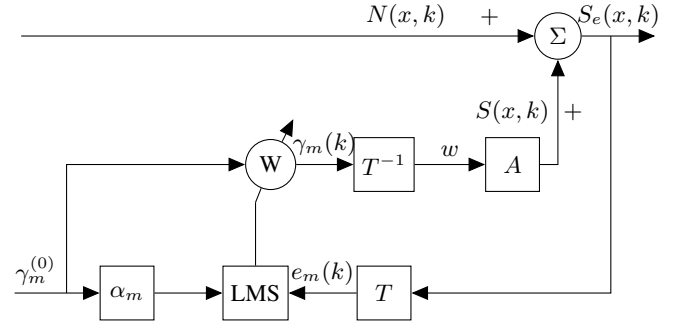


Figure 2: Block diagram of the wave-domain FxLMS algorithm for ANC. Blocks of  $T$  and  $T^{-1}$  represent wave-domain transformation and inverse wave-domain transformation, respectively.

For example, for sound propagation in the free field,  $A(\mathbf{x}|\mathbf{y}, k) = \frac{i}{4} H_0^{(1)}(k\|\mathbf{y} - \mathbf{x}\|)$  and  $\alpha_m(k) = \frac{i}{4} H_m^{(1)}(kr)$ , where  $H_m^{(1)}$  denotes the first kind Hankel function of order  $m$ .

Wave domain noise cancellation is to design the source coefficients  $\gamma_m(k)$  based on the ATF information  $\alpha_m(k)$  so that the noise field  $N(\mathbf{x}, k)$  characterised by its coefficients  $\beta_m(k)$  is completely cancelled within the desired quiet zone. An optimal solution would be  $\gamma_m(k) = -\beta_m(k)/\alpha_m(k)$  if a stable noise field is assumed. However, in practice the noise field is always unknown and could be time-varying, an adaptive algorithm is proposed in the next section.

### 3. ADAPTIVE WAVE DOMAIN PROCESSING

In the proposed system (Figure 1), a single circular microphone array is placed on the boundary of the quiet zone to measure the residual signals. There are  $Q$  microphones equiangularly placed in the error microphone array to measure the sound field  $S_e(\mathbf{x}, k)$ , the superposition of the noise field and sound field generated by the control sources. The error signals in the wave domain are written as

$$e_m(k) = \frac{1}{Q J_m(kr)} \sum_{q=1}^Q S_e(\mathbf{x}_q, k) \exp(-i2\pi m(q-1)/Q), \quad (8)$$

$m = -M, \dots, M.$

It is clear that the number of microphones required is  $Q \geq 2M + 1$  so that all the wave domain error signals can be obtained without causing a spatial aliasing problem.

The wave domain Filtered-X Least Mean Square (WD-FxLMS) algorithm is developed in the following to update the source coefficients for cancelling the noise field over a spatial region. The residual signal in each iteration is expressed as

$$\hat{e}_m(n) = \beta_m + \alpha_m \times \gamma_m(n), \quad (9)$$

where  $\gamma_m(n) = W_m(n) \gamma_m^{(0)}$ ,  $n$  denotes the iteration step,  $W_m(n)$  are the updated weights, and  $\gamma_m^{(0)}$  represents an arbitrary initial estimate of the source distribution coefficients. The wavenumber  $k$  has been omitted for notational simplicity.

The adaptive algorithm minimises the instantaneous squared error,  $\xi_m^{(n)} = |\hat{e}_m(n)|^2 = \hat{e}_m(n) \hat{e}_m^*(n)$ ; and adopting the steepest

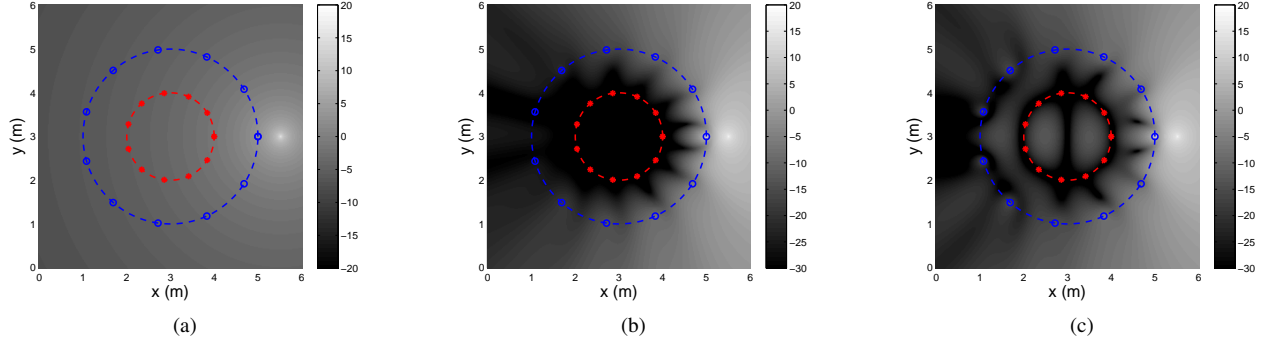


Figure 3: The results of ANC in free-field. The inner array is the microphone array, outer array is the loudspeaker array. (a) The energy of the initial noise field. (b) The residual energy after 30 iterations of WD-FxLMS. (c) The residual energy after 30 iterations of MC-FxLMS.

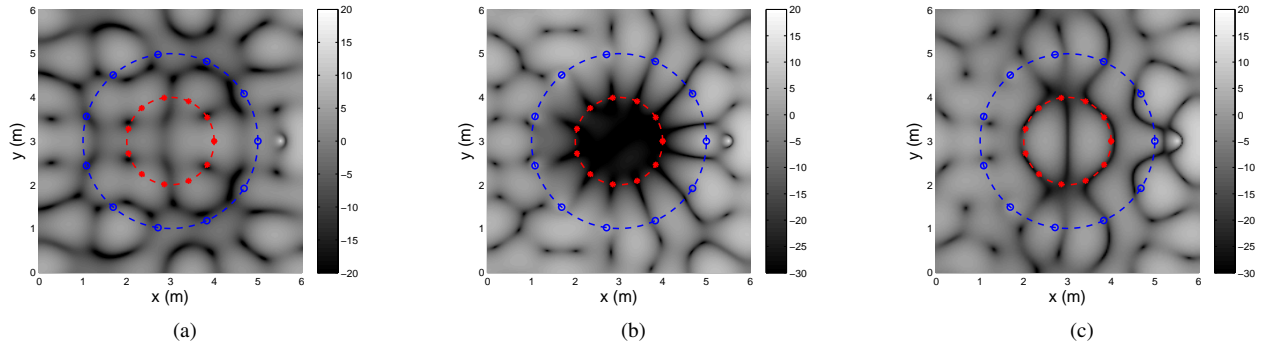


Figure 4: The results of ANC in reverberant environment. The inner array is microphone array, outer array is loudspeaker array. (a) The energy of the initial noise field. (b) The residual energy after 30 iterations of WD-FxLMS. (c) The residual energy after 30 iterations of MC-FxLMS.

descent algorithm, we have

$$W_m(n+1) = W_m(n) - \frac{\mu}{2} \nabla \xi_m(n), \quad (10)$$

where  $\nabla \xi_m(n) = 2\hat{e}_m(n)[\alpha_m \gamma_m^{(0)}]^*$ . Note that here we deal with complex numbers and thus use the complex LMS algorithm to update the weights [25].

Thus, we have the wave domain FxLMS algorithm,

$$W_m(n+1) = W_m(n) - \mu \hat{e}_m(n)[\alpha_m \gamma_m^{(0)}]^*, \quad (11)$$

where the maximum step size  $\mu_{\max} = 1/\|\alpha_m \gamma_m^{(0)}\|^2$  [5]. The block diagram of the algorithm is shown in Figure 2.

Given the estimated source distribution coefficients, a discrete loudspeaker array is designed to approximate the continuous source distribution. For example, we assume  $L \geq 2M + 1$  loudspeakers equiangularly placed in the circular array; the  $\ell$ th loudspeaker driving signal is

$$w_\ell = \rho(\mathbf{y}_\ell, k) \Delta \phi, \quad (12)$$

where  $\Delta \phi = 2\pi/L$ .

#### 4. SIMULATION

In this section, the proposed wave domain adaptive algorithm, WD-FxLMS, is evaluated through simulations of cancelling the noise field due to a 2D point source in a free-field environment and a reverberant environment, respectively. The algorithm is compared

with the conventional frequency domain multichannel noise cancelling algorithm, MC-FxLMS [11]. In the simulation, the quiet zone is a circular region of radius 1 m. We place the loudspeaker array on a circle of  $R = 2$  m and the microphones are equally spaced on the boundary of the quiet zone. A signal-to-noise (SNR) ratio of 40 dB is added to the microphone recordings.

*Case 1—Free-field Environment:* Plots in Figure 3 show the results of cancelling the noise field in free-field environment using the proposed WD-FxLMS and conventional MC-FxLMS. The noise source is a 2D omni-directional point source located at (5.5, 3) and operates at the frequency of 200 Hz. According to (2) and the rule of thumb [23], such a noise field within the desired quiet zone (radius of 1 m) needs  $m \in [-5, 5]$  orthogonal modes to represent; thus, we place 11 loudspeakers and 11 microphones in each corresponding array. After 30 iterations of wave-domain adaptive processing, Figure 3(b) shows that the noise energy inside the entire quiet zone has been reduced to a very low level. Using the same loudspeaker array and microphone array setup, the conventional MC-FxLMS algorithm [11] however can only achieve noise cancellation at the microphone positions or the points very close to the microphones as shown in Figure 3(c). Inside the quiet zone, a fairly high level of noise field still exists. Figure 5 compares the convergence performance of the WD-FxLMS and MC-FxLMS algorithm, where 11 microphone recordings and measurements from 1296 points uniformly placed inside the regions are used. The WD-FxLMS demonstrates fast convergence at both recording positions and at points inside the region.

*Case 2—Reverberant Environment:* The simulations are made

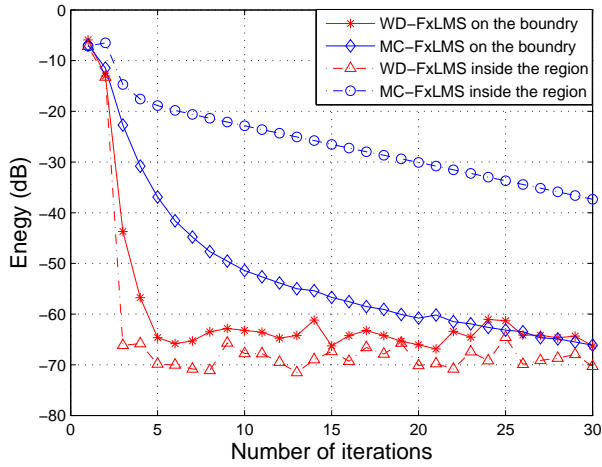


Figure 5: Comparison of convergence performance for noise cancellation using WD-FxLMS and MC-FxLMS algorithm in free-field.

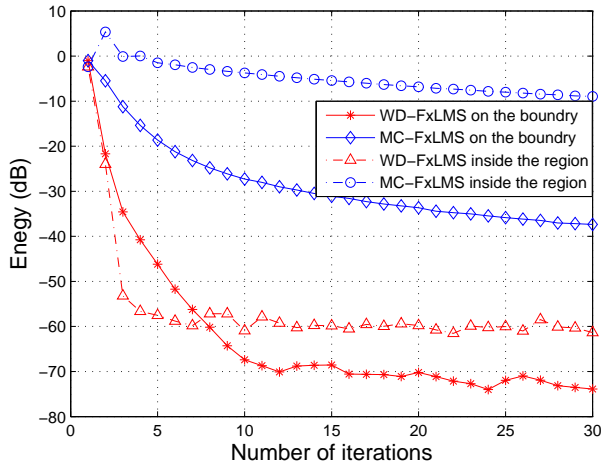


Figure 6: Comparison of convergence performance for noise cancellation using WD-FxLMS and MC-FxLMS algorithm in reverberant environment.

in a room of size 6 m×6 m and all the side walls have a reflection coefficient of 0.9. The reflections are simulated by image source method. The origin of the loudspeaker and microphone arrays is the centre of the room. The same noise source as in the case of free-field environment is simulated. The results in Figure 4 demonstrate that the proposed WD-FxLMS algorithm can cancel the noise field completely within the desired quiet zone; while the conventional MC-FxLMS can only cancel noises at the microphone positions or the boundary of the quiet zone. Figure 6 compares the convergence performance of the WD-FxLMS and MC-FxLMS algorithm. Both can achieve noise cancellation at the recording positions, i.e., the boundary of the designed quiet zone. It can be seen clearly that the convergence rate of the proposed WD-FxLMS is much faster than that of the conventional MC-FxLMS algorithm. For all the points inside the designed region, only the wave domain adaptive

processing can achieve noise energy reduction to a very low level. We can conclude that the proposed WD-FxLMS algorithm is much more effective, especially for achieving noise cancellation over a spatial region.

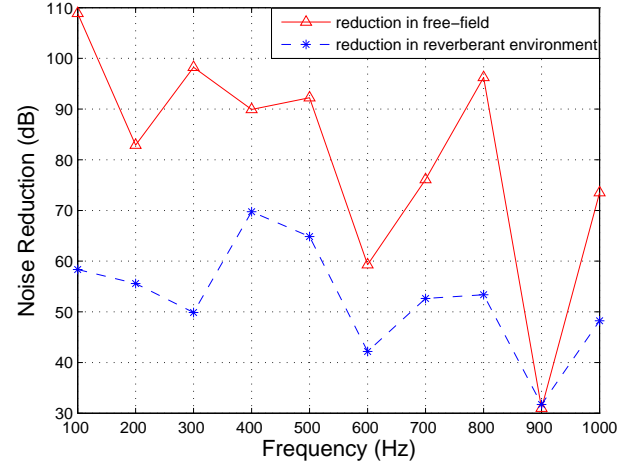


Figure 7: Broadband performance of noise reduction using WD-FxLMS algorithm after 30 iterations.

We next investigate the broadband performance of the proposed algorithm. The system is designed at the frequency of 1 kHz. The noise field within a radius of 1 m is due to 10 noise sources uniformly distributed on a circle 2.5m away from the origin and at angles  $[0 : 36 : 324]^\circ$ . Figure 7 shows that for all the frequencies below 1 kHz, using the WD-FxLMS algorithm, the noise can be reduced significantly within the desired quiet zone. The average noise reduction is around 80 dB and 50 dB, in free-field and reverberant environment, respectively. Notice that the performance is significantly degraded at 600 Hz and 900 Hz. This is due to the fact that at these particular frequencies, the Bessel functions are close to zero (i.e.,  $J_m(kr) \approx 0$ ) and the coefficient error is amplified. This problem could be avoided by placing two closely spaced microphone arrays [16].

## 5. CONCLUSION

Adaptive wave-domain processing was presented in this paper to achieve noise cancellation over a spatial region. We proposed a feedback control system using one microphone array and one loudspeaker array to measure the residual signals and produce anti-noise signals. The system were implemented in both free-field and reverberant environments and compared with the conventional multi-channel ANC system. Simulation results show that the proposed adaptive wave-domain algorithm achieves significant noise reduction over the entire design region with fast convergence speed.

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